ELECTRIC SIMULATOR FOR SOLVING HEAT- AND MASS-TRANSFER PROBLEMS

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UDC 681.142.334

An electric analog simulation method is described for heat- or mass-transfer based on the decomposition of the transport equation and the simulation of a system of locally-one-dimensional equations by resistors.

Analog electric network simulators have been widely used to solve problems of mathematical physics [1].

We consider the locally-one-dimensional method [2, 3] of simulating parabolic equations of multidimensional problems because of its universality and simplicity.

As shown in [2-4] the solution of the equation

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = \sum_{\nu=1}^{p}\left\{\frac{\partial}{\partial\eta_\nu}\left[l_\nu(\eta_\nu,\theta)\frac{\partial\theta}{\partial\eta_\nu}\right] + u(\eta_\nu,\theta)\frac{\partial\theta}{\partial\eta_\nu}\right. \tag{1}$$

can be obtained by solving in succession the p one-dimensional equations

$$\frac{1}{p}\frac{\partial\theta}{\partial \mathrm{Fo}} = \frac{\partial}{\partial\eta_\nu}\left(l_\nu\frac{\partial\theta}{\partial\eta_\nu}\right) + u\frac{\partial\theta}{\partial\eta_\nu} \tag{2}$$

in the time intervals

$$\mathrm{Fo}^{k+\frac{\nu-1}{p}} \leqslant \mathrm{Fo} \leqslant \mathrm{Fo}^{k+\frac{\nu}{p}}. \tag{3}$$

It is convenient to solve Eq. (2) by implicit uniform difference schemes [2, 3]

$$\left(\theta^{k+\frac{\nu}{p}} - \theta^{k+\frac{\nu-1}{p}}\right)/\ \delta\mathrm{Fo} - \Lambda_\nu\theta^{k+\frac{\nu}{p}} = 0. \tag{4}$$

The overall approximation error of a difference scheme is

$$O(\delta\mathrm{Fo}) + O\left(\frac{1}{p}\sum_{\nu=1}^{p} h_\nu^2\right).$$

Scheme (4) can be realized on an electric simulator consisting of resistors (Fig. 1).

The domain of definition of Eq. (1) is replaced by a rectangular network region in which the values of $\theta^{k+1/p}$ are determined successively at nodal points lying on straight lines parallel to η_1. In accordance with the usual methods of simulating one-dimensional problems [1, 5] the points of intersection of these straight lines with the boundaries of the simulator are maintained at potentials E_S or supplied currents I_S corresponding to the boundary conditions at the given instant, and the free ends of the resistors R_{Fo} are maintained at potentials E_0 corresponding to the initial distribution of the values of θ at the given points. By continuing this operation as many times as there are straight lines parallel to η_1 intersecting the region under study we determine $\theta^{k+1/p}$. We then determine $\theta^{k+2/p}$ by using as initial conditions the values of $\theta^{k+1/p}$ at the corresponding points and the conditions at the points where straight lines parallel to η_2

Polytechnic Institute, Donetsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 5, pp. 931-934, May, 1970. Original article submitted July 2, 1969.

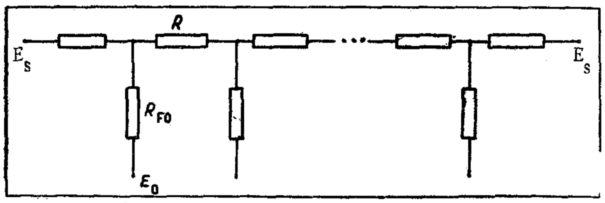

Fig. 1. Circuit diagram of simulator.

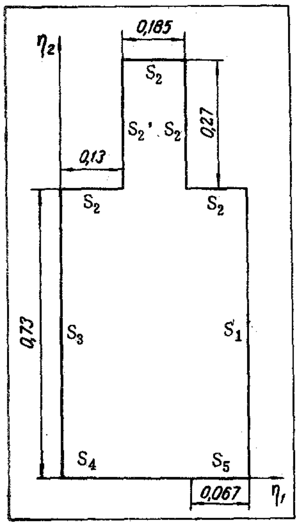

Fig. 2. Schematic diagram of body simulated.

intersect the boundaries etc. The values of θ for integral steps in Fo form the solution of Eq. (1). Since implicit scheme (4) is used in the simulation the choice of δFo is arbitrary and is determined solely by the required accuracy of the solution [1].

Thus rather complicated problems can be solved on a simulator consisting of several tens of variable resistors. This is determined by the possibilities of a discrete one-dimensional electric network simulator [1]. To simulate linear multidimensional heat- and mass-transfer problems simultaneously a two-layer net of resistors of the type used in [6] can be employed.

As an example a two-dimensional heat-conduction problem previously solved on a computer by the method of fractional steps [4] was simulated:

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = \frac{\partial^2\theta}{\partial\eta_1^2} + \frac{\partial^2\theta}{\partial\eta_2^2}. \tag{5}$$

The boundary conditions for the problem (Fig. 2) are the following:

$$\begin{gathered}\theta|_{\mathrm{Fo}=0} = 1; \quad 0 \leqslant \mathrm{Fo} \leqslant 7.42\cdot10^{-2};\\ -\frac{\partial\theta}{\partial\eta_1}\bigg|_{S_1} = \mathrm{Bi}_1\,\theta|_{S_1}; \quad \frac{\partial\theta}{\partial\eta_1}\bigg|_{S_3} = \mathrm{Bi}_3\,\theta|_{S_3};\\ -\frac{\partial\theta}{\partial\eta_2}\bigg|_{S_2} = \mathrm{Bi}_2\,\theta|_{S_2}; \quad \frac{\partial\theta}{\partial\eta_2}\bigg|_{S_4} = \mathrm{Bi}_4\,\theta|_{S_4};\end{gathered} \tag{6}$$

$$\begin{gathered}\frac{\partial\theta}{\partial\eta_1}\bigg|_{S_2'} = \mathrm{Bi}_2\,\theta|_{S_2'}; \quad \frac{\partial\theta}{\partial\eta_1}\bigg|_{S_5} = 0;\\ -\frac{\partial\theta}{\partial\eta_1}\bigg|_{S_2} = \mathrm{Bi}_2\,\theta|_{S_2};\\ \mathrm{Bi}_1 = 1.2; \quad \mathrm{Bi}_2 = 0.366; \quad \mathrm{Bi}_3 = 1.491; \quad \mathrm{Bi}_4 = 0.187.\end{gathered} \tag{7}$$

The locally-one-dimensional scheme corresponding to the problem described by (5) is

$$\frac{\theta^{k+1/2}_{i,j} - \theta^{k}_{i,j}}{2\delta\mathrm{Fo}} = \frac{\theta^{k+1/2}_{i-1,j} - 2\theta^{k+1/2}_{i,j} + \theta^{k+1/2}_{i+1,j}}{h_1^2}, \tag{8a}$$

$$\frac{\theta^{k+1}_{i,j} - \theta^{k+1/2}_{i,j}}{2\delta\mathrm{Fo}} = \frac{\theta^{k+1}_{i,j-1} - 2\theta^{k+1}_{i,j} + \theta^{k+1}_{i,j+1}}{h_2^2}. \tag{8b}$$

We assume as in [4]:

$$jh_2 - \frac{h_2}{2} = 1; \quad h_1 = h_2 = h = 0.0351;$$

$$\delta\mathrm{Fo} = 1.855\cdot10^{-3}.$$

With this choice of mesh size the boundary nodes of the simulator do not correspond to the boundaries of the body. The boundary conditions [4] are used to determine the values of θ at the boundary nodes.

Equations (8a) and (8b) were simulated at each instant k on the same KMS-6 network of variable resistors by using the voltage source and measuring equipment of the EGDA 9/60 integrator. The values of $E^{k+1/2}$ were determined first by successively simulating the horizontal arrays of nodes using the initial conditions (E_0) and the boundary conditions for η_1 = const (Eq. (8a)). Then Eq. (8b) was simulated along vertical lines using as initial conditions the values of $E^{k+1/2}$ at the corresponding nodes and the boundary conditions for η_2 = const. The values of E^{k+1} were then used as initial conditions for the next half-step etc. The solution obtained agreed with the computer solution [4] to within 1%.

Electric simulation by a network of variable resistors using the method of fractional steps is universal and can be used to investigate many problems leading to hyperbolic, biharmonic, and other equations in addition to heat- and mass-transfer problems.

In spite of the fact that a two-dimensional problem can be solved on the simulator described in many fewer steps than on the ÉI-1 static electric integrator, the simulator should be automated to speed up the process. The ÉI-2 instrument developed at Kazakh State University is the first step in this direction [7].

NOTATION

$\theta = (t - t_{min})/(t_{max} - t_{min})$	is the dimensionless temperature;
$Fo = a\tau/\tilde{X}^2$	is the Fourier number;
$\eta_\nu = x_\nu/\tilde{X}^2$	is the dimensionless coordinate;
$\tilde{X}$	is the characteristic dimension of the body;
u	is the dimensionless velocity of the medium;
$l_\nu = \quad \nu/\tilde{\lambda}$	is the dimensionless thermal conductivity of the medium;
$\tilde{\lambda}$	is the characteristic thermal conductivity;
δFo	is the size of the step in Fo;
h_ν	is the step in the coordinate η_ν;
Λ_ν	is the difference approximation for the ν-th operator in Eq. (1);
$Bi = \alpha\tilde{X}/\lambda$	is the Biot number;
E	is the electric potential;
R	is the electrical resistance;
S	is the portion of the surface of the body;
i, j	are the values of spatial variables;
k	is the instant of time.

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